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Question Paper Code : 41363

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Third/Fourth Semester

Electronics and Communication Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Biomedical Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Use of statistical tables are permitted

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If an experiment has the three possible and mutually exclusive outcomes A, B, and C, check in the following case whether the assignment of probabilities is permissible:

   P(A) = 0.57, P(B) 0.24, and P(C) = 0.19.

2. It has been claimed that in 60% of all solar-heat installations the utility bill is reduced by at least one-third. Accordingly, what is the probability that the utility bill will be reduced by at least one-third in four of five installations?

3. The joint probability density function of a bivariate random variable X and Y is

   $$f_{XY}(x,y) = \begin{cases} k(x+y), & 0<x<2, 0<y<2 \\ 0, & elsewhere \end{cases}$$. Determine the value of k.

4. If there is no linear correlation between two random variables X and Y, what can you say about the regression lines?

5. A random process $Y(t)$ consists of the sum of the random process $X(t)$ and a statistically independent noise process $N(t)$. Obtain the cross correlation function of $X(t)$ and $Y(t)$.

6. If $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find whether it is regular.

7. Is it possible for a vector u in a vector space to have two different negatives? Justify.

8. Determine whether the set vectors of the form $(a, b, 1)$ is a subspace of R^3.

9. Prove that the identity $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$ holds for any vectors u and v in an inner product space.

10. Verify that the set of vectors $\{(1,0,-1),(2,0,2),(0,5,0)\}$ is orthogonal with respect to the Euclidean inner product.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Suppose traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.8 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green, what is the probability $P[G2]$ that the second light is green? Also, what is $P[W]$, the probability that you wait for at least one light? Lastly, what is $P[G_1|R_2]$ the conditional probability of a green first light given a red second light? (8)

(ii) The number of messages that arrive at a switchboard per hour is a Poisson random variable with a mean of six. What is the probability for each of the following events:

(1) Exactly two messages arrive within one hour.

(2) No message arrives in one hour.

(3) At least three messages arrive within one hour. (8)

Or

(b) (i) Assume that the length of the phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:

(1) The probability that you will wait more than five minutes before Chris is done with the call.

(2) The probability that Chris call will last between two and six minutes. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by a normal random variable X with mean of 5 and a standard deviation of 4.

(1) What is the probability that a randomly selected parcel weighs between one and ten pounds?

(2) What is the probability that a randomly selected parcel weighs more than nine pounds? (8)

12. (a) A fair coin is tossed three times. Let X be a random variable that takes the value zero if the first toss is tail and the value one if the first toss is head. Also, let Y be a random variable that defines the total number of heads in the three tosses.

(i) Determine the joint probability mass function of X and Y. (6)

(ii) Find the marginal probability distributions of X and Y. (6)

(iii) Are X and Y independent? (4)

Or

(b) (i) The joint probability density function of the random variables X and Y is defined as follows: $f_{XY}(x,y) = \begin{cases} 25e^{-5y}, & 0 \le x \le 0.2, y \ge 0 \\ 0 & elsewhere \end{cases}$.

(1) Find $f_X(x)$ and $f_Y(y)$.

(2) What is the covariance of X and Y? (8)

(ii) The following marks have been obtained by a class of students in two subjects:

X	25	28	35	32	31	36	29	38	34	32
Y	43	46	49	41	36	32	31	30	33	39

Find the linear regression. (8)

13. (a) (i) A random process is defined by $X(t) = K \cos wt$ where w is a constant and K is uniformly distributed between 0 and 2. Determine $E(X(t))$, the autocorrelation function $X(t)$ and the auto covariance of $X(t)$. (8)

(ii) Three persons A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B or to A. Show that the process is a Markovian. Find the transition probability matrix and classify the states. (8)

Or

(b) A random process has sample functions of the form $X(t) = A \cos(wt + \theta)$ where w is a constant, A is a random variable that has magnitude of +1 and -1 with equal probability, and θ is a random variable that is uniformly distributed between 0 and 2π.

Assume that the random variables A and θ are independent.

(i) Is $X(t)$ a wide sense stationary process? (8)

(ii) Is $X(t)$ a mean-ergodic process? (8)

14. (a) Determine whether the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$, where a and b are real, with standard matrix addition and scalar multiplication is a vector space or not. If not, list all aximos that fail to hold.

Or

(b) Find the basis and the dimension of the solution space of homogeneous system $2x_1 + 2x_2 - x_3 + x_5 = 0$; $-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$; $x_1 + x_2 - 2x_3 - x_5 = 0$; $x_3 + x_4 + x_5 = 0$.

15. (a) (i) Find the rank and nullity of the Matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$ (8)

(ii) Let $T : R^2 \to R^3$ be the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$. Find the matrix for the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3, where $u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $u_2 \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. (8)

Or

(b) Apply Gram-Schmidt process to transform the basis vectors $u_1 = (1,1,1)$, $u_2 = (1,1,0)$, $u_3 = (1,0,0)$ of R^3 with the inner product $\langle u, v \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$ into orthonormal basis.